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Tracking Control for Linear Non-minimum-phase Systems Based on A Blocking-zero Hold Function

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ABSTRACT

This paper proposes a four-step tracking control scheme for linear non-minimum-phase systems. First, the poles of the target system have to be placed all at 0 by means of state feedback. Subsequently, a blocking-zero hold function is proposed to transform this non-minimum-phase continuous system into a discretized minimum-phase one. Its system poles are then relocated at desired stable positions again by use of state feedback. Finally, the tracking control is achieved through pole/zero cancellation technique. The effectiveness of the proposed method is evaluated through simulation examples. The simulation has shown convincing results. The inter-sampling behavior is also investigated with satisfactory outcomes.
keywords : Tracking control, Non-minimum-phase, State feedback, Blocking-zero hold function, Pole/zero cancellation

以阻斷零點的持續函數對線性非最小相位系統的循軌控制

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摘要

本研究提出針對線性非最小相位系統一個四階段的循軌控制方案。第一階段，利用狀態回饋使受控系統的所有極點都落在零。第二階段，用所謂阻斷零點的持續函數將此線性非最小相位連續系統轉換成最小相位的離散系統。第三階段，再利用狀態回饋將此離散系統的極點重新定位於預定的穩定位置。第四階段，循軌控制可經由極點/零點對消技巧來獲得。本研究提出的方法的有效性可藉由模擬來驗證。結果顯示：不僅可追蹤到取樣值，連取樣間的行為也有令人滿意的表現。
關鍵詞：循環控制，非最小相位狀態，回饋阻斷零點的持續函數，極點/零點對消。

1. Introduction

Generally speaking, tracking control is more difficult than regulation or set-point problems, in that the reference trajectory is changing with time [6]. Zeros with positive real parts are termed non-minimum-phase (NMP) zeros because they add phase lag to the system [8]. The existence of non-minimum-phase zeros in the plant will limit the achievable gain-bandwidth and exhibit undershoot and/or overshoot in step response [8], [12]. The tracking control for linear non-minimum-phase systems is therefore especially difficult [7], [9]. Some controllers based on internal model are trying to emulate the inverse dynamics of the non-minimum-phase plant, which will certainly result in an internally unstable behavior [5], [9]. Others use preview control [10], [11] to circumvent this problem by assuming the reference input known *a priori*, which is generally not practical, if not impossible. Besides, there exist performance limitations on non-minimum-phase systems with or without preview tracking [3], [4], and [12]. Unfortunately, many applications in engineering such as robots [13] and aircrafts [14] do contain non-minimum-phase dynamics. It will be very useful if one can resolve this control difficulty.

This paper proposes a four-step tracking control scheme for linear time invariant non-minimum-phase systems. First, the poles of the target system have to be placed all at 0 by means of state feedback. Since the limitations introduced by non-minimum-phase zeros are structural [1], a blocking-zero hold function is proposed to transform this non-minimum-phase continuous system into a discretized minimum-phase one. This may be the critical step that distinguishes our approach from other available approaches. Once the system becomes minimum-phase, the usual approaches can be easily applied to achieve tracking control. In our design, its system poles are then relocated at desired stable positions again by use of state feedback. Finally, the tracking control is achieved through pole/zero cancellation technique. Note that since the system is transformed into minimum-phase, the proposed approach does not suffer from the same problems normally associated with non-minimum-phase systems. Our approach can achieve the tracking control without assuming the reference input known or cancelling the unstable zeros of the plant, thus can avoid the problems mentioned above.

The approach is first described in Section 2, where the four steps of the algorithm are depicted in detail. The effectiveness of the algorithm is validated through a simulation example in Section 3. Concluding remarks are made in the final section.

2. Systems Formulation

Consider the tracking control for a linear time invariant system with transfer function

$$P(s)=\frac{y(s)}{u(s)}=\frac{k_C b(s)}{a(s)}, \tag{1}$$

where $P(s)$ has at least one unstable zero, that is, it's a non-minimum-phase system. The system is strictly proper; that is, the order of $a(s)$, n, is larger than the order of $b(s)$, m. The system has a state space representation

$$\begin{aligned}\dot{x}(t)&=Ax(t)+Bu(t),\\ y(t)&=Cx(t),\end{aligned} \tag{2}$$

where $x\in R^n$ is the system state, $u\in R$ the control input, $y\in R$ the measured system output. The control objective is to design a control law such that the output, $y(t)$, will track any given smooth reference input, $y_r(t)$ as closely as possible.

Since the system has an unstable zero dynamics, it is difficult to design tracking control for such systems [7]. In this paper, a four-stage tracking control design is suggested to solve the tracking control problem for a non-minimum-phase system. In this section, it is assumed that the system is of relative degree one, that is, $n-m=1$.
The proposed control structure is shown in Figure 1, which will be elaborated in details below.

2.1. Stage 1: inner continuous state feedback loop

In Stage 1, a continuous state feedback loop is constructed to relocate all the poles of the open-loop system, $P(s)$, to zero. See the inner feedback loop in Figure 1. Thus, the control input to the system is given by

$$u(t)=-K_C x(t)+u_1(t), \tag{3}$$

where K_C is chosen to place all eigenvalues of $A-B\,K_C$ to zero under the controllability assumption of (A, B) pair. The simulation experiences show that this will give better tracking result in the inter-sampling behavior. The system under the inner feedback control loop (3) becomes

$$\begin{aligned}\dot{x}(t)&=(A-BK_C)x(t)+Bu_1(t)\\ &\equiv A_1x(t)+Bu_1(t),\\ y(t)&=Cx(t),\end{aligned} \tag{4}$$

where the eigenvalues of A_1 ($=A-BK_C$) are all at zero. Or equivalently,

$$G_C(s)=\frac{y(s)}{u_1(s)}=C(sI-A_1)^{-1}B=\frac{k_C b(s)}{s^n}, \tag{5}$$

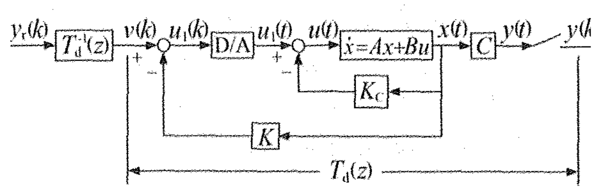


Figure 1: The architecture of the proposed overall control system

2.2. Stage 2: blocking-zero hold function

The key element of the proposed control is a generalized D/A converter in Figure 1, called the blocking-zero hold function. Given the discrete control input $u_1(k) = u_1(t)|_{t=kT}$, a D/A converter generates the continuous control signal $u_1(t)$ from the discrete $u_1(k)$ according to a specific interpolation law.

Notice that since state feedback in Stage 1 does not change the system zeros [5], [7], the controlled system (4) has the same zeros $z_1, \ldots, z_{n-1}$ as those of the uncontrolled system (2). In cases when all zeros are real simple zeros, the interpolation law of the proposed D/A converter is given by

$$u_1(t) = u_1(k)\{1 + m_1[e^{z_1(t-kT)} - 1] + \Lambda + m_{n-1}[e^{z_{n-1}(t-kT)} - 1]\}, \tag{6}$$

where $m_1, \ldots, m_{n-1}$ are design parameters to be determined so that the discrete transfer function from $u_1(k)$ to $y(k)$ has $n-1$ desired stable zeros in the unit circle.

In cases when there are complex conjugate zeros, $z_1 = \sigma + j\omega$, and $z_2 = \sigma - j\omega$, the interpolation law of the proposed D/A converter is

$$u_1(t) = u_1(k)\{1 + m_1 e^{\sigma(t-kT)} \sin(\omega(t-kT)) + m_2[e^{\sigma(t-kT)} \cos(\omega(t-kT)) - 1] + \cdots\}. \tag{7}$$

In cases when there are multiple real zeros, say p multiple zeros at z_1, the interpolation law of the proposed D/A converter is

$$u_1(t) = u_1(k)\{1 + m_1[e^{z_1(t-kT)} - 1] + m_2(t-kT)e^{z_1(t-kT)} + \Lambda + m_p(t-kT)^{p-1}e^{z_1(t-kT)} + \Lambda\}. \tag{8}$$

The design parameters m_i in the D/A converter (6) are determined based on the procedure outlined below. After discretization, the continuous-time inner loop system (4) cascaded with the proposed D/A converter becomes a discrete system

$$x(k+1) = Fx(x) + Gu_1(k), \qquad y(k) = Cx(k), \tag{9}$$

where $u_1(k)$ is the input to the proposed D/A converter in (6), and

$$F = e^{A_1 T}, \tag{10}$$

$$G = \int_0^T e^{A_1\eta}B\{1 + m_1[e^{z_1(T-\eta)} - 1] + \Lambda + m_{n-1}[e^{z_{n-1}(T-\eta)} - 1]d\eta = G_0 + (G_1 - G_0)m_1 + \Lambda + (G_{n-1} - G_0)m_{n-1}, \tag{11}$$

where

$$G_0 = \int_0^T e^{A_1\eta}Bd\eta, \tag{12}$$

$$G_i = \int_0^T e^{A_1\eta}Be^{z_i(T-\eta)}d\eta, \quad i = 1, \ldots, n-1.$$

Without loss of generality, one can assume that the representation (9) is in observer canonical form [7]. Hence the coefficients in the column vector G uniquely determine the zero locations of the discrete transfer function from $u_1(k)$ to $y(k)$,

$$y(k) = G_d(z)u_1(k), \qquad G_d(z) = C(zI - F)^{-1}G = \frac{k_d b_d(z)}{a_d(z)} \tag{13}$$

Since G_i, $i = 1, 2, \ldots, n-1$, in (12) are linearly independent vectors, given any stable locations of the $n-1$ zeros of $G_d(z)$, z_i^d, $i = 1, 2, \ldots, n-1$, there exist unique design constants m_i, $i = 1, 2, \ldots, n-1$, that achieve the zero placement of $G_d(z)$.

The unique feature of the proposed design is that one uses the zero functions $e^{z_i(t-kT)}$ in the interpolation law, where z_i is the zero of the continuous time system. The so-called blocking-zero hold function is then completely determined and should be able to transform the linear continuous system with non-minimum-phase zeros into a discretized one with minimum-phase zeros. It is worth noticing that the determination of these $(m+1)$ parameters depends only on the plant and the desired zeros.

2.3. Stage 3: outer discrete state feedback loop

In the previous Stage, the discretized system (9) has a system matrix $F = e^{A_1 T}$. Since all eigenvalues of A_1 ($= A - BK_C$) are at zero, the eigenvalues of F are all at $\lambda_i(F) = e^{\lambda_i(A_1)T} = e^{0T} = 1$. In other words, the discretized system is not a stable system. Hence, an outer loop using discrete state feedback is constructed to stabilize the discretized system (9),

$$u_1(k) = -Kx(k) + v(k), \tag{14}$$

where $v(k)$ is the auxiliary input coming from the feedforward compensator in Figure 1 and K is the state feedback gain chosen to place all eigenvalues of ($F-GK$) to desired stable locations. Substituting (14) into (9), the closed-loop system formed by the outer feedback loop has the following state space equation

$$x(k+1) = (F - GK)x(x) + Gv(k), \qquad y(k) = Cx(k). \tag{15}$$

The corresponding closed-loop transfer function, $T_d(z)$, from $v(k)$ to $y(k)$ is obtained as follows.

$$y(k) = T_d(z)v(k), \qquad T_d(z) = C(zI - F + GK)^{-1}G = \frac{k_t b_t(z)}{a_t(z)}. \tag{16}$$

2.4. Stage 4: discrete feedforward compensation

The feedforward compensator in Figure 1 is to shape the reference output $y_r(k)$ so that when it is fed into the feedback loop system $T_d(z)$, the system output

$y(k)$ will track the reference output $y_r(k)$ exactly. Since $T_d(z)$ has only stable zeros (according to the Stage-2 design), and stable poles (according to the Stage-3 design), the feedforward compensator is chosen as the direct inversion of $T_d(z)$; that is, the auxiliary input $v(k)$ is given by

$$v(k)=T_d^{-1}(z)y_r(k)=\frac{a_t(z)}{k_t b_t(z)}y_r(k), \qquad (17)$$

which ensures that $y(k)$ tracks the reference $y_r(k)$ asymptotically.

Remark: Each of the four stage designs introduced above has its function. The Stage-3 design is to stabilize the closed-loop system $T_d(z)$ in Figure 1, which is essential to the safe operation of the system. The Stage-2 design is to place all zeros of $T_d(z)$ in the stable region, so that in Stage-4, one can obtain a stable feedforward compensator $T_d^{-1}(z)$. The Stage-4 design guarantees that $y(k)$ tracks $y_r(k)$ asymptotically. Finally, the Stage-1 design and the Stage-2 design contribute to a satisfactory inter-sampling tracking result in the sense that $y(t)$ is close to $y_r(t)$ for all time t.

3. Simulation Example

As an example, consider a third order system with transfer function of the form

$$P(s)=\frac{(s-3)(s-5)}{s^3}.$$

Note that the system is of relative degree one with n = 3, m = 2, and two unstable zeros. The approach proposed in Section 2 can be directly applied. Since the open loop poles are already at 0, there is no need to do the first step state feedback. So $A_1 = A$ and $u_1(t)= u(t)$. That is,

$$\dot{x}(t)=Ax(t)+Bu(t),$$
$$y(t)=Cx(t),$$

where

$$A_1=A=\begin{bmatrix}0&1&0\\0&0&1\\0&0&0\end{bmatrix},\quad B=\begin{bmatrix}0\\0\\1\end{bmatrix},\quad C=\begin{bmatrix}15\\-8\\1\end{bmatrix}.$$

Here, we have chosen to represent the system in control canonical form. The reference input was chosen to be $y_r(k)$ = sin(0.1kT), which is the fundamental component form of Fourier series expansion.

Since there are two open loop zeros involved, the proposed blocking-zero hold function for this system is

$$u_1(t)=u_1(k)\{1+m_1[e^{5(t-kT)}-1]$$
$$+m_2[e^{3(t-kT)}-1]\},$$

where the design parameters, m_1 and m_2, are found to be 0.1356, and -0.9039. With the desired stable zeros at 0.1 and 0.4, the system transfer function from $u(k)$ to $y(k)$ after discretization becomes

$$G_d(z)=\frac{0.4039(z-0.1)(z-0.4)}{(z-1)^3}.$$

Note that although the discretized open loop system indeed becomes minimum-phase as expected, the poles are all at 1, which are not desirable. Subsequently, the state feedback control law is applied to relocate the system poles as desired. In the simulation, desired closed loop poles, are chosen to be 0.9, 0.5, and 0.3. The state feedback gain vector K is found to be [2.4072, 24.3546, 13.0400]. This results in the closed loop transfer function

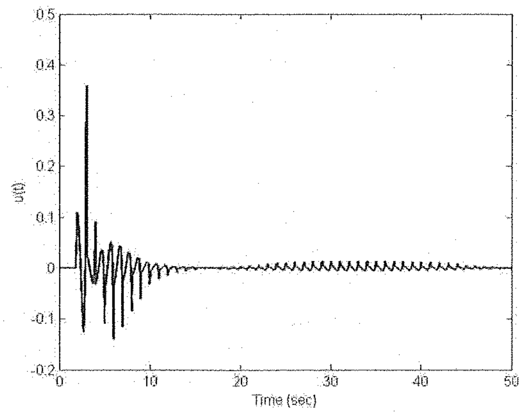
$$T_d(z)=\frac{0.4039(z-0.1)(z-0.4)}{(z-0.3)(z-0.5)(z-0.9)}.$$

Finally, use the pole/zero cancellation techniques to find

$$\frac{v(k)}{y_r(k)}=T_d^{-1}(z)=\frac{(z-0.3)(z-0.5)(z-0.9)}{0.4039(z-0.1)(z-0.4)}.$$

Proper selection of the sampling rate is crucial in computer-controlled systems. Too long a sampling period will have aliasing problem and thus make it impossible to reconstruct the continuous-time signal. Too short a sampling period will increase the load on the computer [2]. Since the period of the reference input, $y_r(k)$, is $\frac{2\pi}{\omega}=\frac{2\pi}{0.1}=62.8319$ sec, the sampling time T, according to sampling theorem, must be smaller than half of its period, *i.e.*, 31.4159 sec in this case.

The simulation was done with sampling time T = 1 sec. The results with step size N = 50 are shown in Figures 2-3. As noted in Figure 2, the control input is very small, *i.e.*, very little effort needed to achieve the tracking. Also as observed in Figure 3, the response is fairly quick and tracks the reference input well. The accuracy is reasonably good with the tracking error of –1.95% in 50 steps. The inter-sampling behavior is relatively smooth without ripples as indicated in a blown-up Figure 4.



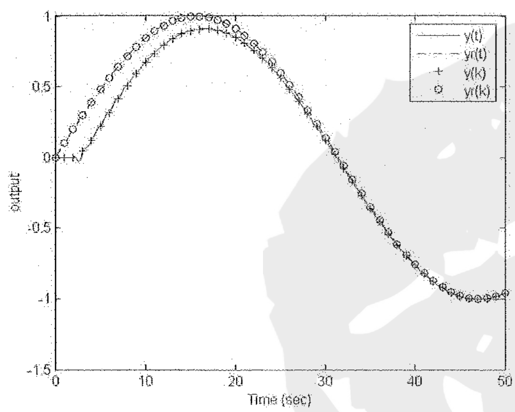


Figure 2: Control Input versus Time

Figure 3: System Output versus Time

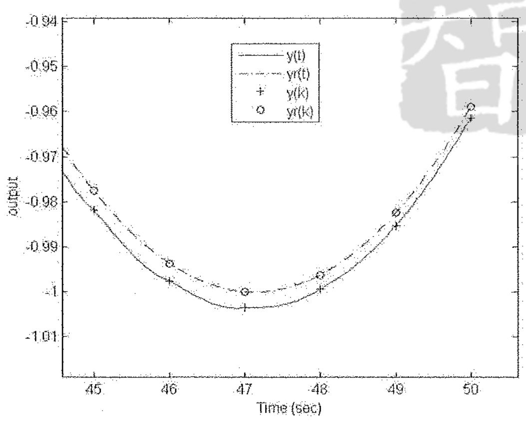


Figure 4: A Blown-up of System Output versus Time

4. Conclusions

This paper proposes a four-step design for tracking control for linear non-minimum-phase systems. Systems with unstable zeros usually render difficulties in tracking control. The system poles are first placed all at 0 by means of state feedback. From our experiences, this will bear better inter-sampling behavior.

Then, through the proposed blocking-zero hold function, the target system is transformed into a discrete minimum-phase system, which is much easier to control. This transformation objective is achieved by determining the design parameters in the blocking-zero hold function to give desired stable zeros in the discrete system. The blocking-zero hold function also makes use of the unique property of blocking zeros to smooth out inter-sampling behavior. This may be the critical step that distinguishes our approach from other available approaches. Once the system becomes minimum-phase, it is not difficult to achieve tracking control.

In our design, state feedback is again employed to relocate the closed sub-system poles at desirable positions. Finally, pole/zero cancellation technique is employed to give unity transfer function for the complete system. In this way, the system output should hopefully track the reference input.

The effectiveness of the proposed design is evaluated through simulation examples. The simulation results have indeed shown quick response and good accuracy. Smooth inter-sampling behavior is also confirmed.

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