ACTA OCEANOGRAPHICA TAIWANICA
SCIENCE REPORTS OF THE NATIONAL TAIWAN UNIVERSITY
NO. 3, PP. 87-96, 5 FIGS, DECEMBER, 1973

# ELEMENTARY WAVE MODEL AND THE DEFINITION OF "FETCH AREA" IN WAVE PREDICTION

NAI KUANG LIANG[1]

## ABSTRACT

It is assumed that a partial area of the wind field can be regarded as an independent generation zone of circular "elementary waves". A "fetch area" is defined, which corresponds to the meaning of fetch and has the same dimension (length). But, fetch area describes it more naturally (Liang, 1973).

## INTRODUCTION

The classical wave theories after Gerstner, Airy and Stokes have treated only two dimensional waves. The significant wave ($H_{1/3}$,$T_{1/3}$), which characterizes wind waves, obeys the small amplitude wave theory (Sverdrup and Munk, 1947). The random wave is the superposition of many sinusoidal small amplitude waves in different directions and different frequencies with random phases (Pierson, 1955).

The wave prediction is an important subject in coastal engineering. There are three factors—wind velocity, wind duration and fetch (deep water). If we assume that the wind is stationary and the wind velocity is given, then only fetch is left. Fetch is originally defined as the length from the interested point to the fartherst point in the wind field in the direction of wind (Fig. 1). This definition is not satisfactory, because we know that the fetch width plays also a roll (Saville, Thorndike, 1954) (Tang, 1965). Like Eckart (1953), it will be assumed that a partial area of the wind field can be seen as an independent generation zone of wind waves.

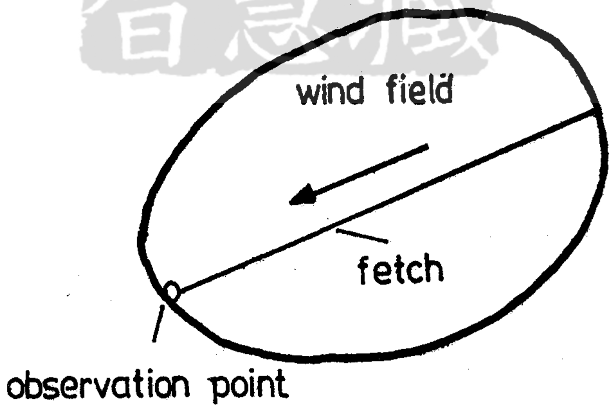


Fig. 1

## ELEMENTARY WAVE MODEL

After Huygens the interference and diffraction of light are declared by the secondary spherical

1. Institute of Oceanography, National Taiwan University

wave hypothesis. If the flapplate of a wave machine is cut into several parts and each part moves in random, each part is then a generation center of circular waves. That means, that the wave has a tendency to become circular wave. By this reason it is assumed that a partial area of the wind field is regarded as an independent generation region. The wave which is directly generated by the wind in this partial area is called "elementary wave". All elementary waves superimpose into the wind wave. This is the elementary wave model. The elementary wave problem is similar to Cauchy-Poisson wave problem (Lamb, 1957), which was solved only two dimensionally. The spreading of the elementary wave energy will be studied under the following assumptions:

1. The elementary wave becomes circular wave during her spreading.
2. The wind is stationary.

A piece of wind field is cut out as an independent generation center of elementary waves (Fig. 2).

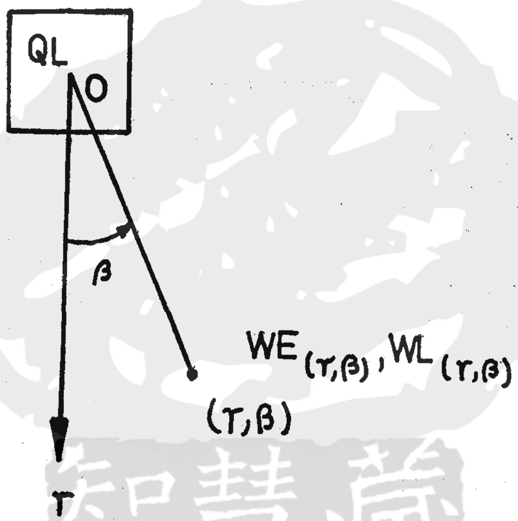


Fig. 2



The average energy per second which is transfered from the wind directly in this cutting is $QL$, $WE_{(r,\beta)}$ is the average elementary wave energy (per unit area) at point $(r,\beta)$. $WL_{(r,\beta)}$ is the average elementary wave power (per unit length) at point $(r,\beta)$. The latter is proportional to $WE_{(r,\beta)}$. The proportionality depends upon the frequency. Using the 1. and 2. assumptions the elementary waves of all frequencies have reached all respected points. Therefore this proportionality is independent from location.

In the book "Water Waves" by Stoker (1957) the potential function of a circular wave is propotional to $1/\sqrt{r}$, if $r$ is not too small, where $r$ is the distance from the origin. Because $\eta=-\frac{1}{g}\left.\frac{\partial\phi}{\partial t}\right|_{\eta}$, where $\eta$ is the wave surface function. So $\eta$ is proportional to $1/\sqrt{r}$. It follows that the elementary wave energy is proportional to $1/r$. It will be studied about the relationship between elementary wave and the angle $\beta$. We define the radiation function $R(\beta)$ for $r=r_1$ and $R'(\beta)$ for $r=r_2$ as follows:

$$R(\beta)=\frac{WE_{(r_1,\beta)}}{WE_{(r_1,0^\circ)}} \tag{1}$$

$$R'(\beta)=\frac{WE_{(r_2,\beta)}}{WE_{(r_2,0^\circ)}} \tag{2}$$

Because $WE$ is proportional to $1/r$ for constant $\beta$,

$$WE_{(r_1,\beta)}=C'\frac{1}{r_1}=WE_{(r_1,0°)}R(\beta) \tag{3}$$

$$WE_{(r_1,\beta)}=C'\frac{1}{r_2}=WE_{(r_2,0°)}R(\beta) \tag{4}$$

and

$$WE_{(r_1,0°)}=C''\frac{1}{r_1} \tag{5}$$

$$WE_{(r_2,0°)}=C''\frac{1}{r_2} \tag{6}$$

We set eq. (5) into eq. (3) and eq. (6) into eq. (4).

$$C'\frac{1}{r_1}=C''\frac{1}{r_1}R(\beta), \quad R(\beta)=\frac{C'}{C''} \tag{7}$$

$$C'\frac{1}{r_2}=C''\frac{1}{r_2}R'(\beta), \quad R'(\beta)=\frac{C'}{C''} \tag{8}$$

It follows

$$R(\beta)=R'(\beta) \tag{9}$$

It means that the radiation function is independent of the radius $r$.
Then we have

$$WE_{(r,\beta)}=C''R(\beta)\frac{1}{r} \tag{10}$$

and

$$WL_{(r,\beta)}=CR(\beta)\frac{1}{r} \tag{11}$$

We consider now a circle with radius $r$. The energy flux, which passes through the circle, is equal to $QL$, which is the energy obtained in $dA$ from wind per second (stationary).

$$QL=\int_S WL_{(r,\beta)}ds=\int_{-\pi}^{\pi} CR(\beta)\frac{1}{r}rd\beta=C\int_{-\pi}^{\pi} R(\beta)d\beta \tag{12}$$

$$C=\frac{QL}{\int_{-\pi}^{\pi} R(\beta)d\beta} \tag{13}$$

let

$$C_1=\frac{1}{\int_{-\pi}^{\pi} R(\beta)d\beta} \tag{14}$$

we have

$$WL_{(r,\beta)}=C_1\frac{1}{r}R(\beta)QL \tag{15}$$

The variability in direction of wave travel after Arthur (1949) corresponds to the radiation. He wrote that the variability might be equal to $\cos^2\beta$, where $\beta$ is the angle to the wind direction. A qualitative experiment is carried out in order to find the radiation function $R(\beta)$.

## EXPERIMENT

The experiment is accomplished in a circular water basin with 380 cm diameter (Fig. 3). An air stream is generated by a wind nozzle, which is about parallel to the water surface. In order to avoid the disturbance from the still air two guiding walls are fastened to the wind nozzle, and a sheet metal is set at the end of the wind field for the limitation of the wind field. The wind field locates at the middle of the basin. The experiment is executed with different constant wind velocities. The reflexion of the wave is kept as small as possible by the pebble bank at edge. After the waves are already stationary, the wave heights are measured at different points, which

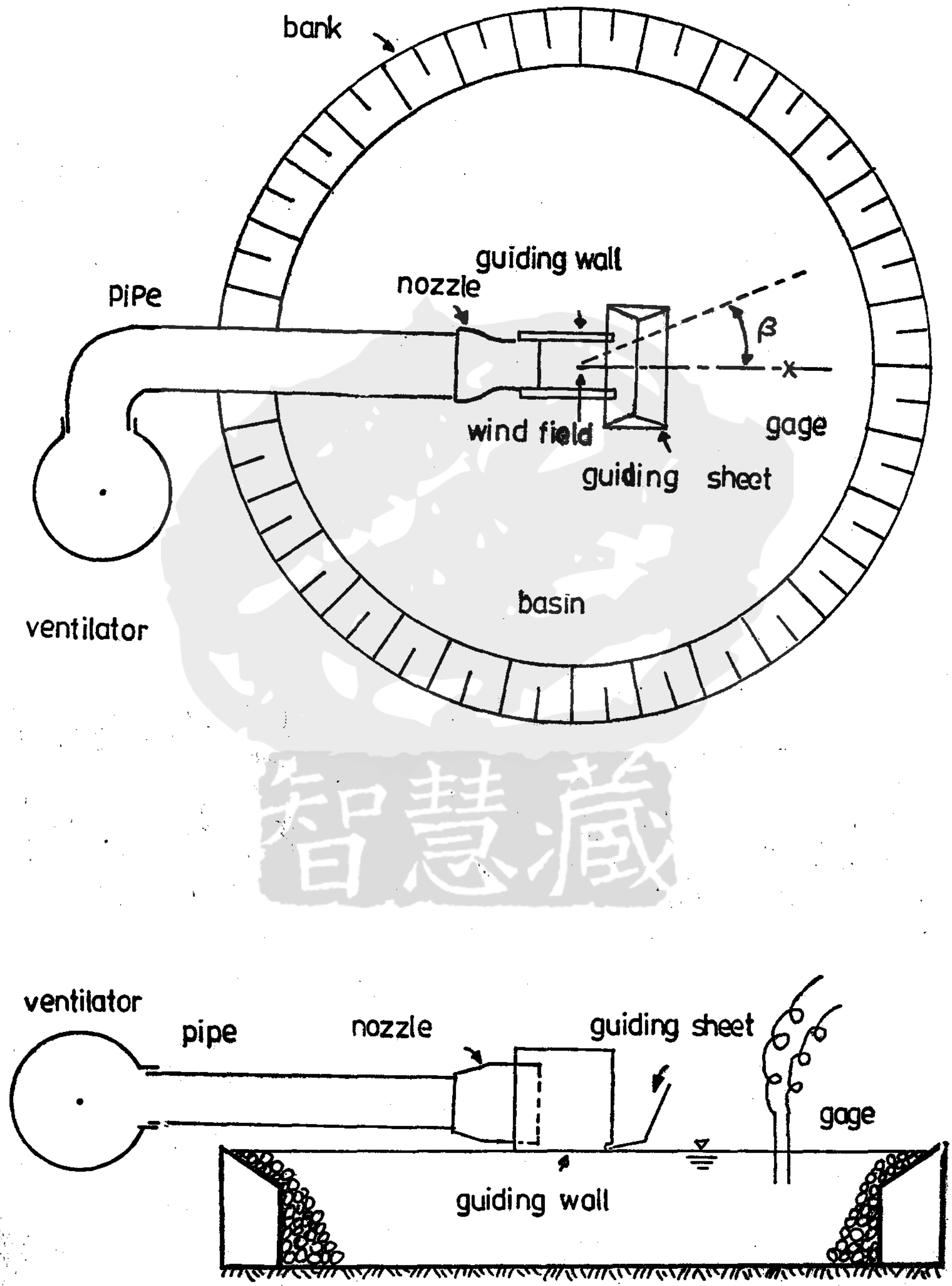
bank
guiding wall
nozzle
PiPe
β
x
wind field
gage
guiding sheet
ventilator
basin
ventilator
pipe
nozzle
guiding sheet
gage
guiding wall



**Fig. 3**

have the same distance from the middle of the basin but locate in various directions. So we can find out the radiation function $R(\beta)$. The waves in ocean are mainly gravity waves. If we want to generate gravity waves in this experiment, the model must be very large. With the following reason we can use a small model:

The difference between the gravity wave and the capillary wave in the wave theory after Airy is that they have different dynamical boundary conditions. Therefore they have different celerity formulae. But the measurement is carried out after the waves are statistical stationary.

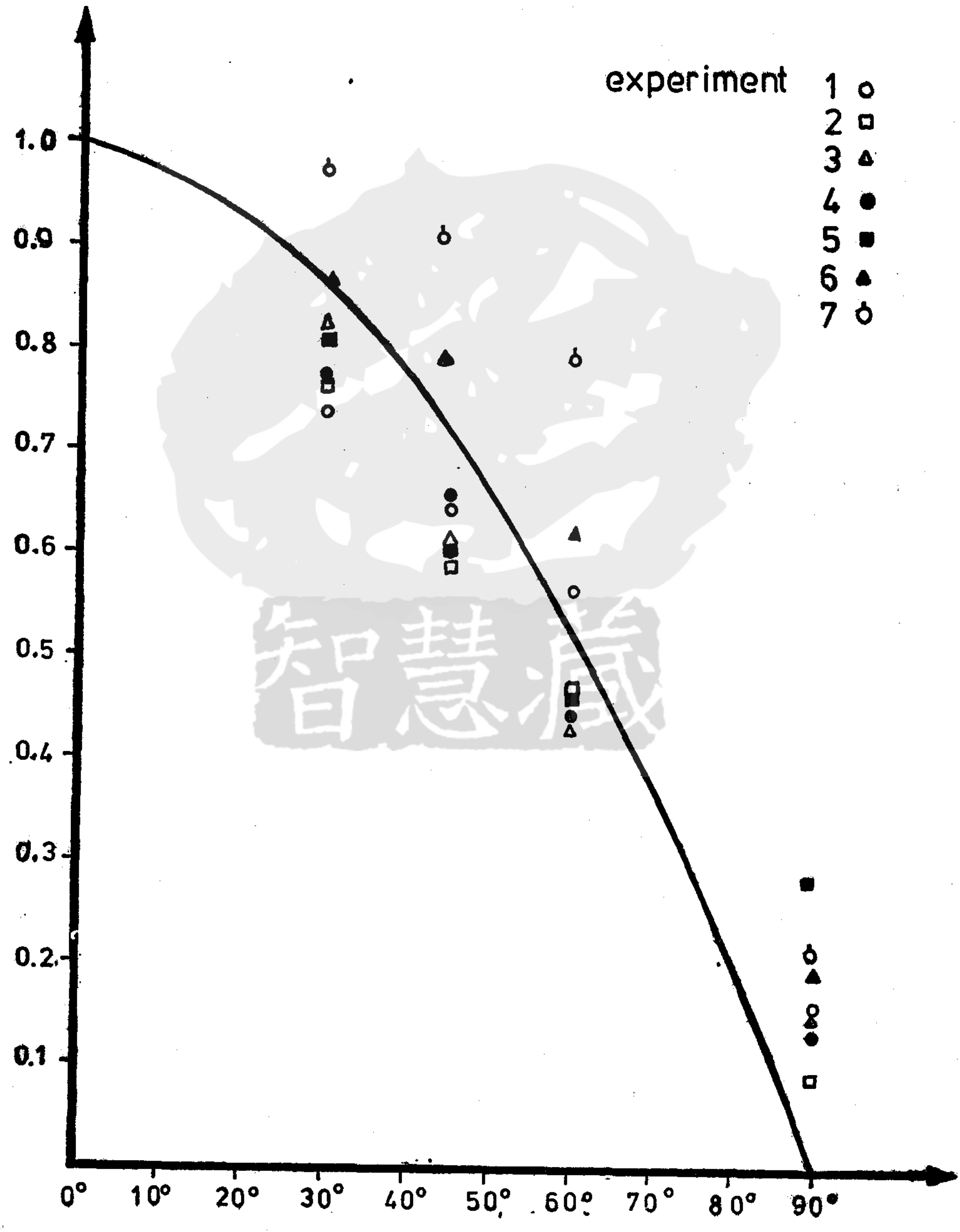


Fig. 4

That means that the celerity has no influence on the experiment and the gravity wave and capillary wave are the same in this experiment.

Because $E \sim H^2_{1/3}$, so $R(\beta)=H^2_{1/3}|_\beta/H^2_{1/3}|_{0°}$ or $\sqrt{R(\beta)}=H_{1/3}|_\beta/H_{1/3}|_{0°}$ The result shows in Fig. 4. The thick curve is $\cos\beta$. We see that the wave energy decreases as the angle increases. At $\beta=90°$ the wave height is small. If we consider wave energy, which is proportional ot the square of wave height, then it is still smaller. We can believe that $R(\beta)=\cos^2\beta$ or the radiation function has an approximation to the square of cosine $\beta$.

## THE DEFINITION OF "FETCH AREA"

The average power $QL$ which is transferred from wind in a small area is still unknown. The interaction between wind and wave is very complicated. Nowadays we know that there are two important effects—one is due to the turbulent fluctuations of the wind (Phillips, 1957) (Eckart, 1953), and the other is the instability of the air stream (Miles, 1957) or the sheltering effect due to the existing wave (Jeffreys, 1925).

The second effect dominates, when the waves are already large. A parameter, transfer coefficient, which will be empirical determined, is derived after Jeffreys sheltering-effect theory and the following assumptions: The waves in a small region are statistical equal and independent from one another. The corresponding mean values and standard deviations are constant.

After Sverdrup and Munk (1947) the pressure increase $\Delta p$ by the wind is as follows:

$$\Delta p=s\rho'(U-c)^2\frac{\partial\eta}{\partial x} \tag{16}$$

$s$ sheltering coefficient
$\rho'$ air density
$U$ wind velocity
$c$ celerity
$\eta$ water surface function
$x$ horizontal coordinate in the direction of wind

The average energy transfer $R$ of one wave length per second per square meter is the following:

$$R=\frac{1}{L}\int_0^L\Delta pv\,dx=\frac{s\rho'}{L}\int_0^L(U-c)^2\frac{\partial\eta}{\partial x}\,\frac{\partial\eta}{\partial t}dx \tag{17}$$

where

$L$ wave length
$v$ the vertical velocity of water particle at the water surface
$t$ time

In elementary wave model the point of observation is fixed. On this account the term $(U-c)^2$ in eq. (17) becomes $U^2$. The transfer power per square meter at one point is:

$$RF=s\rho'\frac{\partial\eta}{\partial x}\,\frac{\partial\eta}{\partial t}\,\bar{U}^2=\xi\bar{U}^2 \tag{18}$$

where $\bar{U}$ is the average wind velocity and $\xi$ is the transfer parameter and equal to $s\rho'\frac{\partial\eta}{\partial x}\,\frac{\partial\eta}{\partial t}$. The transfer parameter is a random variable. We define that $\xi_i$ is the transfer parameter at an arbitary point $i$ in a small area of wind field $dA$. With the above mentioned assumptions and the central limit theorem of statistics the transfer coefficient $\bar{\xi}=\frac{1}{n}\sum_{i=1}^{n}\xi_i$ approaches to constant when $n\to\infty$ as a limit.

From eq. (18)

$$RE_i=\xi_i\bar{U}^2 \tag{19}$$

The average wind-wave energy transfer power per square meter in $dA$ is as follows.

$$\overline{RE} = \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} RE_i = \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \xi_i \bar{U}^2 = \bar{U}^2 \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \xi_i = \bar{\xi}\bar{U}^2 \tag{20}$$

The transfer power in $dA$

$$QL = \overline{RE}\, dA = \bar{\xi}\bar{U}^2\, dA \tag{21}$$

After eq. (15) and eq. (21)

$$WL_{(r,\beta)} = C_1 \frac{1}{r} R(\beta)\, QL = C_1 \bar{\xi} \frac{1}{r} R(\beta) \bar{U}^2 dA \tag{22}$$

We use the polar coordinate (Fig. 5)

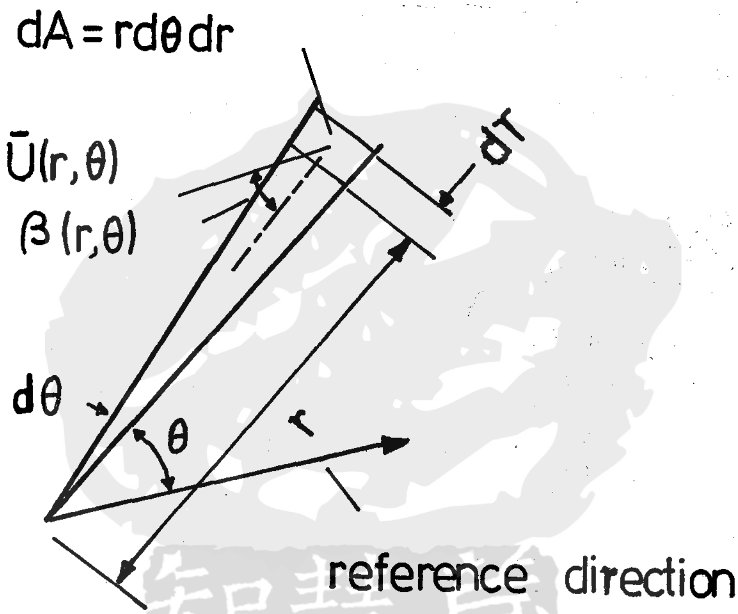


Fig. 5

When the fartherst elementary wave energy has reached the interested point, the wave power (the mean wave energy flux) $LS$ is:

$$\begin{aligned} LS &= \int_{\text{Wind field}} WL_{(r,\beta)} = \int_A C_1 \overline{RE} \frac{1}{r} R[\beta(r,\theta)] dA \\ &= C_1 \int_0 \int_\theta \bar{\xi}(r,\theta)\, \bar{U}^2(r,\theta)\, R[\beta(r,\theta)] dr d\theta \end{aligned} \tag{23}$$

If $\Phi(\sigma)$ is the wave spectrum at the interested point, the average wave energy $E$ is the following.

$$E = \int_0^\infty \Phi(\sigma) d\sigma \tag{24}$$

The wave power $LS$ is then

$$LS = \int_0^\infty \Phi(\sigma) \frac{g}{2\sigma} d\sigma = \frac{g}{2} \int_0^\infty \Phi(\sigma)\sigma^{-1} d\sigma \tag{25}$$

We define the representative group velocity $G$ as follows.

$$G = \frac{LS}{E} = \frac{g}{2} \frac{\int_0^\infty \Phi(\sigma)\sigma^{-1} d\sigma}{\int_0^\infty \Phi(\sigma) d\sigma} \tag{26}$$

Then we have

$$E=\frac{LS}{G}=\frac{C_1}{G}\int_r\int_\theta \bar{\xi}(r,\theta)\cdot\bar{U}^2(r,\theta)\cdot R[\beta(r,\theta)]drd\theta \tag{27}$$

In eq. (27) $\bar{U}^2$ is wind speed $\bar{\xi}$ is a coefficient which should be empirical determined, $G$ is a factor which is dependent from wave spectrum $\Phi(\sigma)$, and $R[\beta(r,\theta)]$ represents the influence of the wave radiation. Because the scalar spectrum $\Phi(\sigma)$ would have some functional relationship (Phillips, 1958), it is expected that the representative group velocity $G$ can be estimated. In eq. (27) we would regard $\bar{\xi}$, $\bar{U}^2$ and $G$ as constants.

We define

$$\text{fetch area} \equiv FF \equiv \int_r\int_\theta R[\beta(r,\theta)]drd\theta \tag{28}$$

"fetch area" corresponds to the meaning of fetch and has the dimension—length $\times$ radian. Because radian is dimensionless, so fetch area has the same dimension as fetch.

## SUGGESTIONS

1. By dimension analysis we often use the dimensionless fetch, $\frac{gF}{\bar{U}^2}$ ($F$ is fetch) and the other dimensionless parameters to analyse wave prediction empirically. I suggest to use $\frac{g\cdot FF}{\bar{U}^2}$ instead of $\frac{gF}{\bar{U}^2}$.

2. A wave generator for short-crested waves in a basin of the hydraulic laboratory would consist of several independent wave generating plates, which move in random.

## CONCLUSIONS

1. The elementary wave model is similar to the theory of the energy balance equation (Hasselmann, 1968). The latter considers also the whole wind field.

2. $R(\beta)=\cos^2\beta$ is only an approximation. Because the radiation function is like a weighted function in the definition of fetch area, an approximation is enough.

3. Fetch area has the same meaning like fetch but describes the problem more naturally.

4. The transfer coefficient $\bar{\xi}$ and the representative group velocity $G$ must be determined empirically by measurements in the ocean in order to get a new method of wave prediction.

## ACKNOWLEDGEMENTS

This research work is sponsered by the german academic exchange service (DAAD). I thank Prof. Dr. -Ing. W. Hensen, former director of FRANZIUS-INSTITUT in the technical University Hannover (West Germany), very much, who has given me the chance to work. I am very indebted to Prof. Dr. -Ing. Partenscky, director of FRANZIUS-INSTITUT, and Prof. Dr. -Ing. Fuehrboeter of the technical University Braunschweig (West Germany), who have given me many important advices. Besides, I thank Mr. Dipl. -Ing. Daemrich, assistant in FRANZIUS-INSTITUT, too.

## REFERENCES CITED

ARTHUR, R. S. (1949) Variability in direction of wave travel, in ocean surface waves. *Ann. N.Y. Acad. Sci.* 51(3): 511-522.

ECKART, C. (1953) The genration of wind waves on a water surface. *Jour. Appl. phys.* 24(12): 1485-1494.

HASSELMANN, K. (1968) Weak interaction theory of ocean waves, basic developements in fluid dynamics, Vol. 2, Academic Press, New York; pp. 117-182.

JEFFREYS, H. (1925) On the formation of water waves by wind. *Proc. Roy. Soc.* A107: 189-206.

LAMB, H. (1957) Hydrodynamics, London, Cambridge University Press.

LIANG, N.K. (1973) Ueber den Einfluss des Windfeldes bei der Wellenvorhersage, Heft 39, Mitteilungen des Franzius-Instituts fuer Wasserbau und Kuesteningenieurwesen der Technischen Universitaet Hannover: pp. 218-323.

MILES, J.W. (1957) On the generation of surface wave by shear flows. *Jour. Fluid Mech.* 3(2): 185-204.

PHILLIPS, O.M. (1957) On the generation of waves by turbulent wind. *Jour. Fluid Mech.* 2(5): 417-445.

PHILLIPS, O.M. (1958) The equilibrium range in the spectrum of wind-generated waves, *J. Fluid Mech.* Vol. 4.

SAVILLE, THORNDIKE, Jr. (1954) The effect of fetch width on wave generation, Tech. Mem. No. 70 Beach Erosion Board, Corps of Engineers.

STOKER, J.J. (1957) Water waves, Interscience Publishers, Inc. New York.

SVERDRUP, H.U. and W.H. Munk (1947) Wind, sea and swell: theory of relation for forecasting. U.S. Navy Hydrographic Office Public No. 601: 44 pp.

TANG, L.W. (1965) Report on the wave forecasting for the design of Penghu Bay Bridge. Bulletin No. 10. The Tainan Hydraulic Laboratory, Cheng Kung Univ., Taiwan, China.

# 元素波模式與「吹風面積」在風浪推算中之定義

梁 乃 匡

## 摘 要

假設風域中任一小區可視為圓形「元素波」生成之中心，由此推出「吹風面積」之定義，其與吹風距離之意義相同並且有相同之單位（長度），但它說明得更真實些。