

# The Grey Complex Number

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## ABSTRACT

Grey system theory is a new subject to treat the thing whose information is poor or missing and grey complex number is one of important parts of grey mathematics. We give the definition of grey complex number and get its complex number-covered set, the whitened complex number and the only potential true complex number. Then some computational rules are also obtained. The proposed definitions and results can enrich the contents of grey mathematics.

**Keywords:** Grey system theory, Grey complex number, Grey mathematics, Number covered set.

## 1. Introduction

The Grey system theory aims at the uncertain situation with poor or missing information ([1-4]). The grey situation is general in reality for the complexity of real systems, and it makes that grey system theory becomes a popular methodology. For the missing information, we could not get the true value but its boundary. The unknown value is a grey number and the boundary is its number-covered set [4]. Because grey system theory owns many virtues regarding to the information-missing situation, it has been successfully applied in many fields [4-12]. But it also encountered many difficulties because the mathematical foundation is imperfect [13]. In order to establish the grey mathematics, we get some basic definitions and results [2-4]. Then the grey complex number is also proposed in this paper and we get its computational rules.

The rest of this paper is organized as follows: In

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section II, we introduce the complex number. Then the grey complex number is proposed and some results can be obtained in section III. At last, the conclusions and future works are pointed out.

## 2. Preliminaries

In order to propose the grey complex number, we introduce the complex number and it can be seen in many mathematical textbooks.

Let  $i$  be the imaginary unit, where  $i^2 = -1$ . Generally, there are two types of complex number:

1. The general form:  $z = a + bi$ , where  $a$  and  $b$  are real numbers and called as the real and imaginary parts of the complex number  $z$ , respectively;

There are some computational rules for the general form

- (1)  $a + bi = c + di \Leftrightarrow a = c, b = d$
- (2)  $(a + bi) \pm (c + di) = (a \pm c) + (b \pm d)i$
- (3)  $(a + bi)(c + di) = (ac - bd) + (ad + bc)i$
- (4)  $\frac{a + bi}{c + di} = \frac{ac + bd}{c^2 + d^2} + \frac{bc - ad}{c^2 + d^2}i$

2. The triangular form:

$$z = a + bi = r(\cos\theta + i\sin\theta)$$

where  $r = \sqrt{a^2 + b^2}$  is the norm of the complex number  $z$ , and  $\theta$  is the main angle of  $z$ , which should satisfy the following equations

$$0 \leq \theta < 2\pi \text{ and } \tan\theta = \frac{b}{a}.$$

supposing that

$$z_j = r_j(\cos\theta_j + i\sin\theta_j), \quad j = 1, 2,$$

then we also have the following computational rules:

- (1)  $z_1 = z_2 (\neq 0) \Leftrightarrow r_1 = r_2, \theta_1 = \theta_2$
- (2)  $z_1 z_2 = r_1 r_2 [\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)]$
- (3)  $\frac{z_1}{z_2} = \frac{r_1}{r_2} [\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]$

On the other hand,  $a - bi$  is called as the conjugated complex number of  $a + bi$ .