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ON THE STEADY STATE CURRENTS IN SHALLOW SEAS OR LAKES

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ABSTRACT

A wind-driven current model in a shallow water, which considers bottom condition of non-zero velocity, is developed. A slip parameter γ is introduced to relate bottom stress and velocity on the bottom. It is found that the model will be reduced to the Welander's model if γh is greater than 100. h is the depth of water. When the model was applied to Lake Superior, it was revealed that the bottom condition (i. e., slip or no-slip) does affect vertical distribution of the horizontal velocity on or near the bottom of the water body, but not the circulation pattern.

INTRODUCTION

The classical theory for the sea level changes produced in a deep sea by a steady wind stress was developed by Ekman (1923). Welander (1956) has later on extended the theory to a shallow sea in which the depth of frictional influence is comparable to the depth of the sea. Gedney and Lick (1972) then applied the Welander's results to calculate steady state current profile in Lake Erie. Bonham-Carter and Thomas (1973) used the same approach to predict the three-dimensional pattern of steady wind-driven currents in Lake Ontario. In these works, the current velocities on the bottom were assumed to be zero. The similar condition was used by Liggett and Hadjitheodorou (1969) and Lin and Perez (1971) to study current distributions in a shallow lake. Liggett (1969), Forristall (1974), and Haq and Lick (1975) used zero bottom current velocity condition in their unsteady state models. However, the current velocities may not be necessary to be zero on the bottom. On the other words, the water particles may slip on the bottom. Such an ideal has been proposed by Birchfied (1967). It is the purpose of this paper to find an analytical solution along the line of Welander's work (1956) with that the current velocities on the bottom are different from zero. The solution will also be discussed under what conditions the solution will be reduced to the Welander's results.

ANALYTICAL RESULTS

To describe a sea level changes in steady state under the wind action on the sea, Welander used the following two equations of motion:

$$
\begin{aligned}
\varepsilon_v \frac{\partial^2 u}{\partial z^2} + fv &= g\frac{\partial \eta}{\partial x} \\
\varepsilon_v \frac{\partial^2 v}{\partial z^2} - fu &= g\frac{\partial \eta}{\partial y}
\end{aligned}
\qquad (1)
$$

with the boundary conditions

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