Terr. Atmos. Ocean. Sci., Vol. 23, No. 4, 439-450, August 2012
doi: 10.3319/TAO.2012.03.28.02(A)

A New Parallel Domain-Decomposed Chebyshev Collocation Method for Atmospheric and Oceanic Modeling

Yu-Ming Tsai[1], Hung-Chi Kuo[1, *], Yung-Chieh Chang[2], and Yu-Heng Tseng[1]

[1] Department of Atmospheric Sciences, National Taiwan University, Taipei, Taiwan
[2] Department of Applied Mathematics, National Chiao Tung University, Hsinchu, Taiwan

Received 19 July 2011, accepted 28 March 2012

ABSTRACT

Spectral methods seek the solution to a differential equation in terms of series of known smooth function. The Chebyshev series possesses the exponential-convergence property regardless of the imposed boundary condition, and therefore is suited for the regional modeling. We propose a new domain-decomposed Chebyshev collocation method which facilitates an efficient parallel implementation. The boundary conditions for the individual sub-domains are exchanged through one grid interval overlapping. This approach is validated using the one dimensional advection equation and the inviscid Burgers' equation. We further tested the vortex formation and propagation problems using two-dimensional nonlinear shallow water equations. The domain decomposition approach in general gave more accurate solutions compared to that of the single domain calculation. Moreover, our approach retains the exponential error convergence and conservation of mass and the quadratic quantities such as kinetic energy and enstrophy. The efficiency of our method is greater than one and increases with the number of processors, with the optimal speed up of 29 and efficiency 3.7 in 8 processors. Efficiency greater than one was obtained due to the reduction the degrees of freedom in each sub-domain that reduces the spectral operational count and also due to a larger time step allowed in the sub-domain method. The communication overhead begins to dominate when the number of processors further increases, but the method still results in an efficiency of 0.9 in 16 processors. As a result, the parallel domain-decomposition Chebyshev method may serve as an efficient alternative for atmospheric and oceanic modeling.

Key words: Chebyshev collocation method, Domain-decomposition, 2-D nonlinear shallow water model

Citation: Tsai, Y. M., H. C. Kuo, Y. C. Chang, and Y. H. Tseng, 2012: A new parallel domain-decomposed Chebyshev collocation method for atmospheric and oceanic modeling. Terr. Atmos. Ocean. Sci., 23, 439-450, doi: 10.3319/TAO.2012.03.28.02(A)

1. INTRODUCTION

Advances in spectral transform methods (Orszag 1970) and Fast Fourier Transform (FFT) brought about successful global atmospheric modeling using these methods (e.g., Bourke et al. 1977; Machenhauer 1979). Compared with finite difference and finite element (volume) methods, global spectral models with spherical harmonics can eliminate the singularity of poles and preserve high accuracy and efficiency due to the "exponential convergence" property and the easy implementation of the semi-implicit method. The spectral methods also allow the discrete conservation of kinetic energy and enstrophy, which are important for two-dimensional turbulence.

In addition to the popularity of spectral methods for global models, the modified Fourier method is often used in operational regional spectral models, such as the forecast model at the Japan Meteorological Agency (Segami et al. 1989), US NCEP regional spectral model (Juang and Kanamitsu 1994; Juang 2000), and the HIgh-Resolution Limited-Area Model (HIRLAM) in Europe (Haugen and Machenhauer 1993). However, the exponential convergence property in these models may reduce to algebraic convergence with $O(N^{-3})$, which loses the major advantage of the Fourier method. Also, the basis functions used in the modified Fourier method need to be determined by the time-dependent boundary conditions and cannot be selected arbitrarily (Roache 1978; Kuo and Williams 1992; Adcock 2009).

* *Corresponding author*
E-mail: kuo@as.ntu.edu.tw